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On the Local Structure of the Temperature Field in a Turbulent Flow,
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The study of the local structure of hydrodynamic fields in turbulent flows at high Reynolds numbers was initiated in 1941 by A. N. Kolmogoroff (1,2) and by A. M. Obukhov (3), by investigating the question of the local structure of the velocity field in an incompressible fluid. As the quantitative characteristics of the structure of this field in the works (1–3), there was used the structural function of the velocity field

$$D_{ll}(r) = \overline{\left[v_l(M') - v_l(M)\right]^2}, \; D_{nn}(r) = \overline{\left[v_n(M') - v_n(M)\right]^2} \qquad (1)$$

where $v_l(M)$ and $v_l(M')$ are the projections of the velocity vector at the points M and M' in the direction $\overrightarrow{MM'}$; $v_n(M)$ and $v_n(M')$ are the projections of the velocity vector in a direction perpendicular to $\overrightarrow{MM'}$; the line above serves as symbol of the mean value and r is the distance between the points M and M' (basis of this is that for large Reynolds numbers for not too great distances r the quantities D_{ll} and D_{nn} essentially are considered as depending only on r, see (1)). In the work (1) the behavior of the functions $D_{ll}(r)$ and $D_{nn}(r)$ are investigated with the aid of dimension theory and similarity theory in (2), for this end, the equation of motion is used and finally, in (3) is used the equation of energy balance in the spectra of the turbulent stream obtained from the motion equation by applying Fourier transformations[1]. It is essential that the results, obtained by these three methods, agree; at present these results are also confirmed by experiment (7, 8).

Recently, A. M. Obukhov (9) applied the development in (1) to a general representation of the local structure of turbulent streams at high Reynolds numbers also and to the question of the local structure of the temperature fields in such streams[2]. The method of investigating the structure function of the temperature field in (9) coincides with the method applied in (1) to study the structure of the velocity field - namely - dimension and similarity theories. In the present note we show that the results[3] obtained in (9) may be verified and somewhat extended with the aid of the equations of hydrodynamics.

As in (9), we consider here only the case when there is in the stream considerable turbulence of dynamic origin and the temperature pulsations are so small that their influence on the development of turbulence may be neglected, and we will not take into account radiant heat exchange in the medium and the heat connected to the dissipation

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[1]This latter method of investigating the structure of velocity fields, in recent years, was repeatedly explained with no essential variation by a series of authors (4-6).

[2]Application of these same representations to the investigation of pressure and acceleration fields in turbulent flows, see (10, 11).

[3]Let us note that these results have already found important applications in the theory of dissipation of light and radio waves in atmosphere (12, 13).

of mechanical energy of turbulence. For these conditions the velocity field of the stream will satisfy the usual motion equations related to an incompressible fluid (mean velocity, we consider, essentially, much less than the speed of sound) and the temperature field by the equation

$$\frac{\partial T}{\partial t} + \sum_{j=1}^{3} v_j \frac{\partial T}{\partial x_j} = \chi \Delta T \qquad (2)$$

(T is the temperature, v_1, v_2, v_3 are the components of the velocity vector, χ is the coefficient of molecular temperature conduction of the medium.) Let us note that equation (2) coincides exactly with the equation of diffusion of certain ingredients in a medium if only by T the concentration of this ingredient is understood and by χ its molecular coefficient of diffusion. In connection with this, all the subsequent reasoning (the same, certainly, as all the reasoning contained in (9)) is related not only to the temperature field but to the field of concentration of any ingredients suspended or dissolved in a turbulent flow.

In the case of large Reynolds numbers turbulent motion may be considered locally homogeneous and locally isotropic (see (1)). Here, from the assumption of (2) it follows that

$$\frac{\partial}{\partial t}\left[\overline{T(M)T(M')}\right] = 2 \sum_{j=1}^{3} \frac{\partial}{\partial \xi_j}\left[\overline{v_j(M)T(M)T(M')}\right] + 2\chi\Delta \left[\overline{T(M)T(M')}\right] \quad (3)$$

where ξ_1, ξ_2, ξ_3 are components of the vector $\overrightarrow{MM'}$ and

$$\Delta = \frac{\partial^2}{\partial \xi_1^2} + \frac{\partial^2}{\partial \xi_2^2} + \frac{\partial^2}{\partial \xi_3^2}$$

(see, for example, the derivation of the analogous equation for the velocity field in the book of L. D. Landau and E. M. Lifshitz (14) page 119). Using now the continuity equation div V=0, we may express the right side of equation (3) by the structural formula

$$D_{lTT}(r) = \overline{\left[v_l(M') - v_l(M) \right] \left[T(M') - T(M) \right]^2} \qquad (4)$$

$$D_{TT}(r) = \overline{\left[T(M') - T(M) \right]^2} \qquad (5)$$

depending only on r[4]. Here we arrive at the equation

$$\frac{\partial}{\partial t} \overline{\left[T(M)T(M') \right]} = \frac{1}{2} \sum_{j=1}^{3} \frac{\partial}{\partial \xi_j} \left[D_{lTT}(r) \frac{\xi_j}{r} \right] - \chi \Delta D_{TT}(r) \qquad (6)$$

Let us put r=0 in this equation; then the first term goes to zero and we obtain the relation

$$\frac{\partial}{\partial t} \overline{\left[T(M)^2 \right]} = -3\chi D''_{TT}(0) \qquad (7)$$

defining the velocity of the equalizing temperature (or concentration) field T(M) and completely analogous to the well known equation defining the velocity of the dissipation of kinetic energy. The role of the mean energy dissipation for unit time on unit mass of fluid ϵ here plays the constant

$$N = \frac{3}{2} \chi D''_{TT}(0) \qquad (8)$$

[4] The continuity equation is necessary here in order to be able to express the mean value $\overline{v_l(M)T(M)T(M')}$ by the structural function $D_{lTT}(r)$. Here must be used the theorem that in an incompressible fluid the correlation of the isotropic velocity field with any isotropic scaler field is always identically equal to zero.

The quantity N was introduced in the reasonings of A. M. Obukhov (9).. Since the function $D_{TT}(r)$ is independent of t (see (1)), then from equations (5), (6) and (7) easily follows the relation

$$2N = -\frac{1}{2}\left(\frac{dD_{lTT}(r)}{dr} + \frac{2}{r} D_{lTT}(r)\right) + \chi\left(\frac{d^2 D_{TT}(r)}{dr^2} + \frac{2}{r}\frac{dD_{TT}(r)}{dr}\right) \qquad (9)$$

connecting the structural function $D_{lTT}(r)$ and $D_{TT}(r)$. Integrating this relation with respect to r and taking into account the condition $D_{lTT}(0) = \frac{dD_{TT}(0)}{dr} = 0$, we obtain the equation

$$D_{lTT}(r) - 2\chi \frac{dD_{TT}(r)}{dr} = -\frac{4}{3} Nr \qquad (10)$$

analogous to the equation of A. N. Kolmogoroff (2)

$$D_{lll}(r) - 6\nu \frac{dD_{ll}(r)}{dr} = -\frac{4}{5}\epsilon r \qquad (11)$$

connecting the structural functions $D_{ll}(r)$ and

$$D_{lll}(r) = \left[v_l(M') - v_l(M) \right]^3$$

Equation (8) shows that for small r in equation (10) it is possible to neglect the first term compared to the second; for such r

$$D_{TT}(r) \approx \frac{1}{3\chi} Nr^2 \qquad (12).$$

For large r, conversely, we may neglect in (10) the second term which describes the molecular temperature conduction (diffusion), so that for large r

$$D_{lTT}(r) \approx -\frac{4}{3} Nr \qquad (13)$$

Let us introduce now in the reasoning the dimensionless quantity

$$F = \frac{D_{lTT}(r)}{D_{TT}(r)\ \sqrt{D_{ll}(r)}} \tag{14}$$

For large Reynolds numbers it is essential to consider that for large r the quantity F is constant, that is, independent of r[5]. But from (13) and (14) it quickly follows that

$$D_{TT}(r) = -\frac{4}{3F}\,Nr\left[D_{ll}(r)\right]^{-\frac{1}{2}} \tag{15}$$

From this equation it is seen, in particular, that the constant F is always negative: $F = -|F|$. Using now the well known "2/3 law", from the works (1–3), for the velocity field

$$D_{ll}(r) = C\epsilon^{2/3}r^{2/3};\ C = \left(-\frac{4}{5S}\right)^{2/3},\ S = \frac{D_{lll}(r)}{[D_{ll}(r)]^{3/2}} \tag{16}$$

we obtain for sufficiently large r, as found by A. M. Obukhov (9) from considerations of the law of dimensionality, the "2/3 law" for the temperature field (or concentration):

$$D_{TT}(r) = k^2 N\epsilon^{-1/3} r^{2/3} \tag{17}$$

where

$$k^2 = -\frac{4}{3F}\left(-\frac{5S}{4}\right)^{1/3} \tag{18}$$

Here the dimensionless constant k^2 of Obukhov we expressed as having the simple probability-theoretical sense of the constants S and F.

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[5]Generally speaking F may depend on the internal Reynolds number $Re^* = \frac{(v^2)^{1/2}r}{\nu}$. By assuming, however, that for $Re^* \to \infty$ the function $F(Re^*)$ tends to the limiting value F_∞, we may replace $F(Re^*)$ by the constant F_∞ in our case of very large Reynolds numbers of the flow for all sufficiently large r.